DOI: https://doi.org/10.59503/29538009-2023.1.11-5

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OPTIMAL ALLOCATION OF PENSION FUNDS. SENSITIVITY ANALYSES OF THE ADJUSTED SHARPE RATIO

Abstract: In order to optimally manage pension funds, an optimal portfolio is constructed under various constraints. The Sharpe and adjusted Sharpe ratios are calculated. An assessment of the symmetry and spread coefficients of the optimal portfolio yield is made. A sensitivity analysis of the optimal portfolio is performed and founded that when using the Sharpe ratio as a decision criterion, the optimal limit for the country and asset is 35%, while under RA law it is 15%, and the obtained result is stable with respect to symmetry and spread.

Keywords: pension funds, optimal portfolio, risk, Sharpe ratio, adjusted Sharpe ratio

JEL code: G28

Research aim: The aim of the research is to achieve maximum efficiency of the pension fund investments by using the Sharpe ratio as a criterion, and show that the result is stable with respect to symmetry and spread.

Research hypothesis: It is supposed that optimal allocation of pension funds and sensitivity analyses is vital for achieving maximum efficiency of the pension fund.

Research novelty: The scientific novelty is the analysis of eligible asset classes for pension funds under looser boundary conditions than imposed limits by law based on the mean-variance portfolio optimization model where Sharpe ratio is taken as a decision criterion and the result is stable with respect to symmetry and spread. The proposed analysis can be used in optimal management of pension funds for asset allocation purposes.

Introduction

There are various approaches by which an investor can evaluate the performance of a portfolio. The main objective of pension fund managers is to manage the optimal portfolio under various asset investment constraints, the asset allocation strategy, whether the weights are optimally allocated, what amount of risk comes from those asset groups, and whether there are assets that diversify the risk of the portfolio. In order to evaluate the performance of the portfolio, the Sharpe and adjusted Sharpe ratio are calculated in the work. Investments also require considering the symmetry and spread of the portfolio distribution, for which the adjusted Sharpe ratio is calculated. Seema Sharma in 'Sharpe ratio and information ratio: do they really help investment decisions?' research [1] thought that simplicity of ratio leads to its greatest weakness. The ratio is useful for evaluating the portfolio but only in limited situation. It is not applicable in asymmetric situations. M. Venugopal and S. Sophia concluded [2] that adjusted Sharpe ratio is the best performance measure that can be used in the volatile markets.

Sharpe ratio has become part of the canon of modern financial analysis, the results of Francisco Barillas, Raymond Kan, Cesare Robotti and Jay Shanken study [3] presented in article suggest that a more sophisticated approach to interpreting Sharpe ratio is called for, one that incorporates information about the investment style that generated the returns and the market environment in which those returns were generated.

The most common and classic model, Markowitz [4] model, is used for optimal asset management. According to Markowitz's approach, in choosing the optimal portfolio, the return and volatility of the portfolio are important, where the amount of risk is the standard deviation of the expected return.

The below is presented the expected return and standard deviation of the portfolio:

$$\mu_p(w) = w^T * \mu, \qquad \sigma_p(w) = (w^T * \Sigma * w)^{0.5},$$

where:

μ - Nx1 vector, i-th element is the expected rate of return on the relevant asset,

w - Nx1 vector, i-th lement is the weight of the corresponding asset in the portfolio and $\sum w = 1$,

μ_p and σ_p - portfolio return and standard deviation,

Σ - NxN covariance matrix of asset returns.

The Markovitz model is considered as the basis for the evaluation of the optimal portfolio. In this study, the following interpretation of the Markovitz model's problem solution with several limitations are considered.

$$\Pi(w) = w^T * \mu - \lambda * w^T * \Sigma * w,$$
$$A * w = b,$$
$$C * w \leq d,$$

where λ is the risk tolerance coefficient.

The portfolio corresponding to the maximum Sharpe ratio is considered as the optimal portfolio [5]. Typically, the Sharpe ratio is used to measure a portfolio's performance, and the higher the ratio, the higher the portfolio's return per unit of risk, where risk is measured by the portfolio's standard deviation.

The Sharpe ratio is:

$$SR = (\mu_p - r)/\sigma_p,$$

where r is the risk-free rate of return.

Investors' risk preference can be described in terms of expected return and standard deviation. For any level of risk, the portfolio with the highest expected return is preferred, or, equivalently, for any level of expected return, the portfolio with the least risk is preferred. The higher the SR ratio, the higher the expected additional return ($\mu_p - r$) for the unit risk (σ_p) of the portfolio. The Sharpe Ratio tells an investor what portion of portfolio's performance is associated with risk-taking. It measures a portfolio's added value relative to its total risk [6].

The Sharpe ratio does not take into account the symmetry and spread of the distribution of the returns of the portfolio, so in order to evaluate the stability of the obtained results, the adjusted Sharpe ratio is calculated in the study. First, the distribution moments of the 3rd and 4th order are calculated, and then the adjusted Sharpe ratio - ASR [5].

$$ASR = SR\left[1 + \frac{\mu_3}{6} * SR - \frac{\mu_4 - 3}{24} * SR^2\right],$$

where μ_3 , μ_4 are the standardized central moments of the corresponding return distribution?

The third-order moment or symmetry factor is a measure of the distribution's symmetry. A distribution is symmetric when the right and left sides are equal, that is, the third-order central moment is

zero. The fourth-order moment, or coefficient of spread, shows the concentration of the distribution around the mean. If the latter is smaller than 3, we have a distribution centered around the mean, and the opposite is a mean of greater spread.

The solution of the problem is presented not analytically, but numerically, using the Matlab program. Under various constraints, optimal portfolios are constructed and by applying the Sharpe ratio, the risk/return behavior of these portfolios are analyzed according to the specified criterion. For the construction of the portfolio, unhedged returns are first calculated, based on the returns on assets in dram value. The optimal portfolios are constructed by applying restrictions by the currency of the asset traded and restrictions by country. Considering that there are missing data in the asset price series, the expected return and the covariance matrix are estimated using the missing data method, and then the covariance matrix is filtered based on the random matrix theory. Next, the optimal portfolio is evaluated based on the Sharpe ratio. For the optimal portfolio, an assessment of risk statistics is carried out by currency, by country and by sector, as well as in the case of various restrictions, by calculating the Sharpe and adjusted Sharpe ratios, a comparative analysis of the portfolios and the stability of the obtained result is performed.

In order to evaluate the optimal portfolio, investments in ETFs [7] / ETF (Exchange traded fund) - a group of bonds, securities and other types of securities / and RA government bond indices / ETF and indices - henceforth active / are considered. The data are observed monthly from 01.01.2014 to 02.28.2022. for the period and are downloaded from Bloomberg [8]. ETFs are selected based on the following criteria: monthly price availability, Morningstar

Performance Rating 4 and 5 stars, region: USA, UK, France, Germany, capital market value greater than 100,000, 3-month average volume greater than 10,000, turnover the coefficient is greater than 1. As a result, 48 ETFs are obtained with the above parameters, and TBI discount bonds with a maturity of 1 week and more and G5I coupon bonds with a maturity of 5 years and more are considered as the index of RA government bonds [9]. In the case of the base scenario, restrictions by country and currency are defined as follows: as lower limit minimum 5% per country and 10% per currency as upper limit 30% per country and 40% per currency maximum investment in each asset 20% and minimum 0%. The return on the risk-free asset is estimated at 8% at work.

As a result of optimization, based on the expected return and risk estimates for the above mentioned assets, the effective limit of the portfolios is calculated: Figure 1. The picture also shows the optimum Sharpe portfolio:

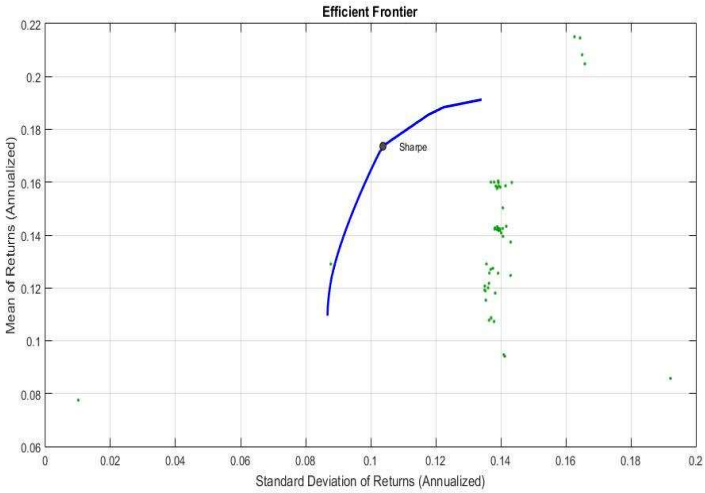

Figure 1. Efficient frontier. Sharpe ratio 0.26

Below shows the distribution of optimal Sharpie portfolio weights by sector and country.

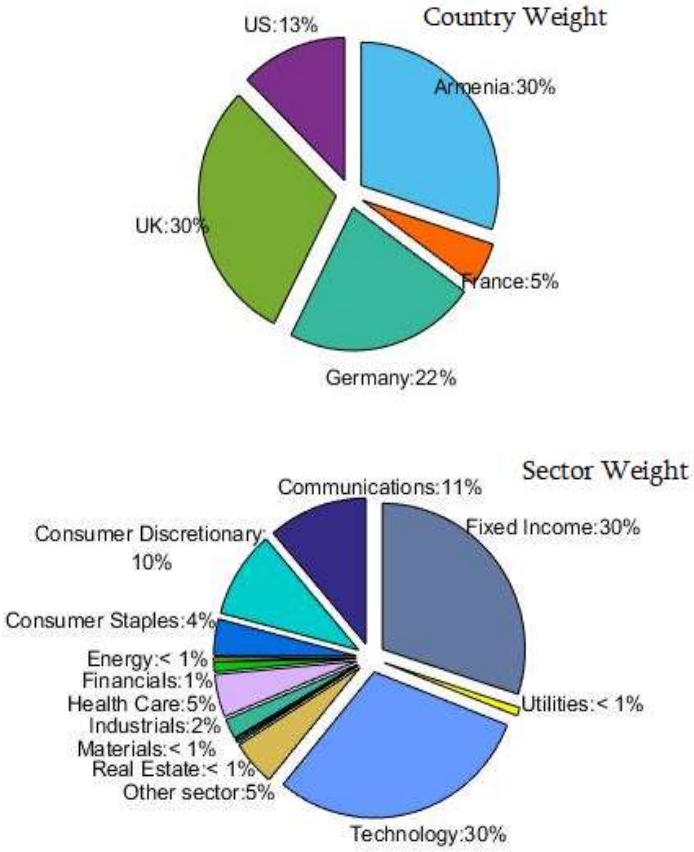

Figure 2. Distribution of Sharpe portfolio weights by country and sector

In order to evaluate the sensitivity of the obtained results to the change of restrictions, as a base option, the restrictions on the investment of assets of the funds defined in the RA Law on Accumulated Pensions [10] are considered, and under these

restrictions, the 3rd and 4th order moments are first calculated /Figure 3/, Sharpe and adjusted Sharpe ratios /Figure 4/. Then the deviations from these restrictions and the sensitivity of the coefficient to them are considered.

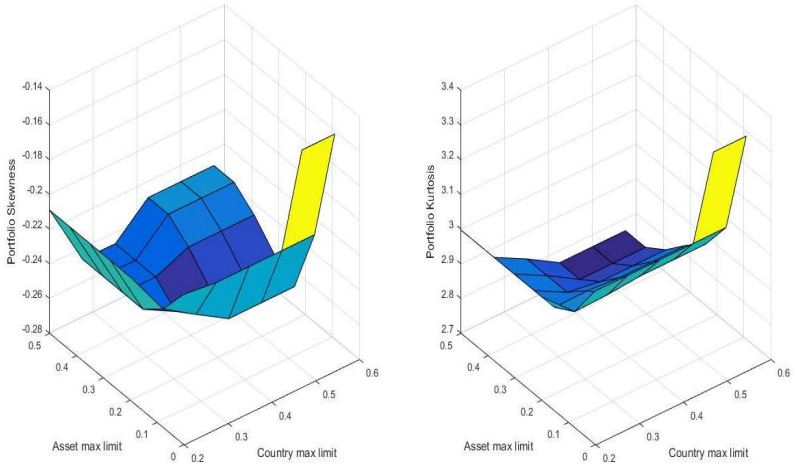

Figure 3. Portfolio skewness and kurtosis

In Figure 3 we received information about the symmetry and distribution of the portfolio; the central moment of the third order is negative, but, taking into account its magnitude, it can be concluded that the distribution of optimal portfolios is symmetric about the mean under the various constraints. Similarly, the plot shows that the fourth-order central moment of the optimal portfolio is very close to three, from which we can conclude that the distribution of returns of the optimal portfolio has almost zero excess (relative to the normal distribution) spread. Therefore, it can be concluded that the distribution of returns of the optimal portfolio does not have significant asymmetry and extra spread.

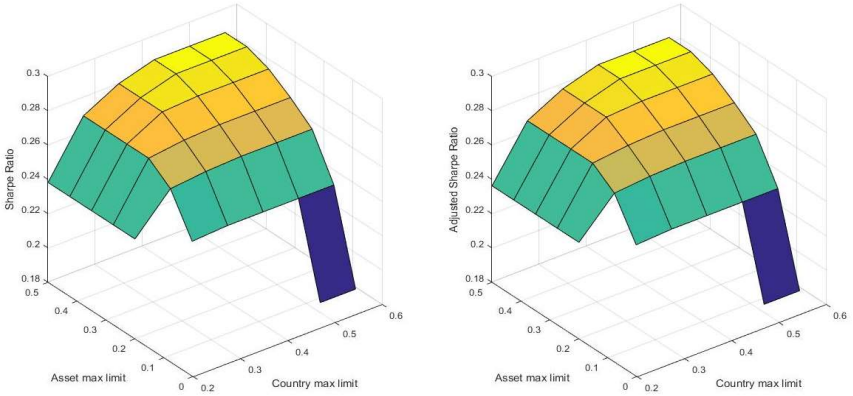

Figure 4. Sharpe and adjusted Sharpe ratios under constraints, sensitivity

As a result, we can see from Figure 4 that the obtained optimal result has no deviations, it is stable with respect to symmetry and spread. Depending on the limitations, the values of the Sharpe ratio are different. A diversified portfolio can be chosen under different constraints, and the more the constraints are relaxed, the larger the Sharpe ratio becomes.

The sensitivity analysis of the Sharpe ratio of the optimal portfolio is carried out depending on the maximum permissible weights of the asset and the country. The range of the maximum weight of the asset is 5…50%, and the range of the maximum weight of the country is 5…60%. As can be seen from Figure 3, the Sharpe ratio of the optimal portfolio reaches its stable level at a maximum limit of 35% of the asset and country, and the change of the Sharpe ratio is almost zero at larger maximum allowable weights. If the Sharpe ratio is 0.23 in the case of restrictions allowed by law, it can reach 0.28 if the restrictions are relaxed, where the optimal restriction is 35% for both the country and the asset. As a result of

the sensitivity analysis of the adjusted Sharpe ratio, it is found that the values of the symmetry of the portfolio range from -0.2294 to -0.1535, that is, the distribution is slightly left-skewed, the probability of accepting values smaller than the mean is greater. Portfolio spread values range from 2.7654 to 3.3267, which means the distribution does not have much extra spread. As a result of the assessment of the corrected Sharpe ratio, it is obtained that the coefficient values are in the range from 0.1850 to 0.2833, and in the case of the Sharpe ratio, from 0.1859 to 0.2862. Therefore, we can conclude that the obtained optimal result is stable with respect to the criterion.

Conclusion

Under the constraints, an optimal portfolio is constructed and the Sharpe ratio is estimated for the optimal portfolio. Then, sensitivity analysis of Sharpe and adjusted Sharpe ratio to different combinations of restrictions by country and by asset is carried out. As a result of the analysis, it is found that when using the Sharpe ratio as a decision criterion, the optimal limit for the country and asset is 35%, while under RA law it is 15%, and the obtained result is stable with respect to symmetry and spread, because the deviations of the values are insignificant.

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ԿԵՆՍԱԹՈՇԱԿԱՅԻՆ ՖՈՆԴԵՐԻ ՕՊՏԻՄԱԼ ՆԵՐԴՐՈՒՄՆԵՐԻ ԿԱՌՈՒՑՈՒՄ. ՃՇԳՐՏՎԱԾ ՇԱՐՊԻ ԳՈՐԾԱԿՑԻ ԶԳԱՅՈՒՆՈՒԹՅԱՆ ՎԵՐԼՈՒԾՈՒԹՅՈՒՆ

Բանալի բառեր – կենսաթոշակային ֆոնդ, օպտիմալ պայուսակ, ռիսկ, Շարպի գործակից, ճշգրտված Շարպի գործակից

Կենսաթոշակային ֆոնդերի օպտիմալ ներդրման նպատակով տարբեր սահմանափակումների ներքո կառուցվել

է օպտիմալ պայուսակ, գնահատվել է օպտիմալ բաշխվածությունն ըստ արժույթի, ըստ երկրի և ըստ սեկտորի: Հաշվարկվել են Շարպի և ճշգրտված Շարպի գործակիցը, կատարվել է պայուսակների համեմատական վերլուծություն: Կատարվել է օպտիմալ պայուսակի եկամտաբերության սիմետրիկության և փռվածության գործակիցների գնահատում: Ըստ երկրի և ըստ ակտիվի սահմանափակումների տարբեր, կոմբինացիաների ներքո իրականացվել է օպտիմալ պայուսակի Շարպի և ճշգրտված Շարպի գործակցի զգայունության վերլուծություն: Վերլուծության արդյունքում ստացվել է, որ Շարպի գործակցի՝ որպես որոշման չափանիշի, կիրառման դեպքում օպտիմալ սահմանափակումը երկրի և ակտիվի դեպքում կազմում է 35%, մինչդեռ ՀՀ օրենքով այն կազմում է 15%, և ստացված արդյունքը կայուն է սիմետրիկության և փռվածության նկատմամբ, քանի որ արժեքների շեղումները աննշան են:

Submitted: 15.04.2023
Accepted for publication: 30.04.2023